ONLINE APPENDIX

Effects of unanticipated trade shocks on sectoral outputs and wages

In this Appendix, we show how trade shocks may affect wages and create inequality in a small open economy with imperfect labor mobility.

We begin with the benchmark case of perfect labor mobility and show how trade shocks affect outputs and factor incomes. Afterwards, we modify the model by assuming imperfect labor mobility.

1. A benchmark 3-sector model with perfect labor mobility

The home country (called Home for short) is a small open economy: it takes international prices as given. The rest of the world is called "The Foreign Country" or simply Foreign. Home produces three goods: good X, good M and good N. Good N is a non-traded good, while goods X and M are international traded. We assume perfect competition. Goods produced by different countries are perfect substitutes. Assume that in equilibrium, Home exports good X and imports good M. The international prices (in US dollars) in Foreign are P_X^* and P_M^*. Trading between Home and Foreign involves positive trade costs (such as inspection costs, bureaucratic delays, transport costs). These are denoted by $\tau_X > 1$ and $\tau_M > 1$, such that Home producers of good X only receive P_X (in US dollars) per unit exported, where $P_X = P_X^*/\tau_X < P_X^*$, and Home consumers must pay a price of P_M (in US dollars) per unit of the imported good, where $P_M = \tau_M P_M^* > P_M^*$.

Import-competing firms in Home can sell good M to Home's consumers at the price P_M (in US dollars). Similarly, Home producers of good X can (if they wish) sell their goods in the Home market at the price P_X (in US dollars). The firms in the tradeable goods sector in the Home Country thus face the price ratio π, defined by

$$\pi = \frac{P_X}{P_M} = \frac{P_X^*}{\tau_X \tau_M P_M^*}.$$

Since P_X^* and P_M^* are given, a fall in either τ_X or τ_M (or both) will increase π. In the home market, there are three price ratios: π, p and q, where p denotes the domestic price ratio of good X to good N, and q denotes the price ratio of good M to good N. The domestic price ratio of exportables to importables is $p/q = \pi$. We assume that in the non-traded good industry the marginal product of labor is constant, and is equal to 1. It follows that in Home's non-traded good sector, the wage rate, measured in terms of the non-traded good, is $W = 1$.The other two sectors, producing good X and M respectively, use sector-specific capital K_X and K_M and mobile labor, L_X and L_M. The total labor force is L, and full employment implies that $L_N + L_X + L_M = L$.[1]

Let Q_N, Q_M and Q_X denote Home's output of goods N, M and X respectively. We assume that the production functions are

$$Q_N = L_N, \tag{1.1}$$

$$Q_M = F^M(L_M, K_M), \tag{1.2}$$

$$Q_X = F^X(L_X, K_X). \tag{1.3}$$

where $F^X(.)$ and $F^Y(.)$ are neoclassical productions with the usual properties. We assume that K_M and K_X are in fixed supply and these variables will be omitted in what follows. Thus we will use the following notations:

$$Q_X = f(L_X) \text{ and } Q_M = g(L_M),$$

where $f(.)$ and $g(.)$ are strictly concave and increasing.

Assume that in equilibrium the outputs and labor inputs in all three sectors are strictly positive. Then, under perfect labor mobility (i.e., the wage rates in all the industries are equalized), production efficiency implies that $pf'(L_X) = W$, and $q\, g'(L_M) = W$, where $W = 1$. Given p and q, we can determine the allocation of labor across sector by using the following equations:

$$pf'(L_X) = 1 \rightarrow \frac{dL_X}{dp} = -\frac{f'}{pf''} = -\frac{1}{p^2 f''} > 0, \tag{1.4}$$

$$qg'(L_M) = 1 \rightarrow \frac{dL_M}{dq} = -\frac{g'}{qg''} = -\frac{1}{q^2 g''} > 0. \tag{1.5}$$

Note that

$$\frac{p}{q} = \frac{g'(L_M)}{f'(L_X)}. \tag{1.6}$$

From (1.4) and (1.5), we get

$$Q_X = Q_X(p), \text{ and } Q_X'(p) = f'(L_X)\frac{dL_X}{dp} = -\frac{1}{p^3 f''} \tag{1.7}$$

[1] These assumptions are used in a number of well known papers (e.g., Grossman and Helpman, "Protection for Sale", *American Economic Review*, 84(4),1994, pp. 833-850).

$$Q_M = Q_M(q),\ \text{and}\ Q'_M(q) = -\frac{1}{q^3 g''} \tag{1.8}$$

$$L_N = \overline{L} - L_X(p) - L_M(q),\ Q_N = L_N$$

and national income in terms of good N is

$$\begin{aligned} Y &= \left(\overline{L} - L_X(p) - L_M(q)\right) + pQ_X(p) + qQ_M(q) \\ &= \overline{L} + [pQ_X(p) - L_X(p)] + [qQ_M(q) - L_M(q)] \end{aligned} \tag{1.9}$$

On the demand side, we assume that consumers' utility function is quasi-linear:

$$U(C_M, C_X, C_N) = u(C_M) + v(C_X) + C_N \tag{1.10}$$

where $u(.)$ and $v(.)$ are strictly concave functions. The representative consumer maximizes U subject to

$$pC_X + qC_M + C_N = E,$$

where E is total expenditure (in terms of good N). The Lagrangian is

$$L = u(C_M) + v(C_X) + C_N + \lambda \left[E - pC_X - qC_M - C_N\right].$$

Assuming an interior solution. The FOCs are

$$u'(C_M) = \lambda q,$$

$$v'(C_X) = \lambda p,$$

$$1 = \lambda.$$

Thus we obtain the demand functions

$$C_M = u'^{-1}(q) \equiv D_M(q) \text{ with } D'_M(q) = \frac{1}{u''(C_M)} < 0, \tag{1.11}$$

$$C_X = v'^{-1}(p) \equiv D_X(p) \text{ with } D'_X(C_X) = \frac{1}{v''(C_X)} < 0, \tag{1.12}$$

$$C_N = E - qD_M(q) - pD_X(p).$$

In equilibrium, the non-traded market must clear, i.e., $C_N = Q_N$. We assume that the total consumption expenditure equals income, $E = Y$. As is well known, this assumption implies that trade balance is zero. To see this, note that conditions $E = Y_N$ and $C_N = Q_N$ imply

$$pD_X(p) + qD_M(q) = pQ_X(p) + qQ_M(q), \tag{1.13}$$

i.e.,

$$[D_M(q) - Q_M(q)] = \frac{p}{q}\left[Q_X(p) - D_X(p)\right],$$

i.e.,

$$M(q) - \frac{p}{q}X(p) = 0, \tag{1.14}$$

where $M(q)$ is the imports demand function, $M(q) \equiv D_M(q) - Q_M(q)$, and $X(p) \equiv Q_X(p) - D_X(p)$ is the export supply function.

Effects of trade shocks on domestic relative prices

Let us show that in our model, an increase in π (which may be caused by a decrease in τ_M or in τ_X, or an increase in the exogenous foreign price ratio, P_X^*/P_M^*) always leads to a fall in q (the price of importables relative to the price of the non-traded good). Using $p = q\pi$, we write the trade balance condition (1.14) as

$$G(\pi, q) \equiv M(q) - \pi X(q\pi) = 0. \tag{1.15}$$

This equation implies that q is a function of the exogenous parameter π. Applying the implicit function theorem to (1.15), we obtain

$$\frac{dq}{d\pi} = -\frac{G_\pi}{G_q} = \frac{[\pi X'(q\pi)q + X]}{M'(q) - \pi^2 X'(q\pi)} = \frac{pX'(p) + X}{M'(q) - \frac{p^2}{q^2}X'(p)} < 0. \tag{1.16}$$

Thus, an increase in π always reduces q, the price of importables in terms of the non-traded good.

To express this result in terms of various elasticities, let us re-arrange eq. (1.16) to get

$$\begin{aligned}
\frac{dq}{d\pi} &= \frac{\left[\frac{pX'}{X} + 1\right]}{\left[\frac{M'}{X} - \left(\frac{p}{q^2}\right)\left(\frac{pX'}{X}\right)\right]} = \frac{\left[\frac{pX'}{X} + 1\right]}{\left[\frac{\pi M'}{M} - \left(\frac{p}{q^2}\right)\left(\frac{pX'}{X}\right)\right]} \\
&= \frac{\left[\frac{pX'}{X} + 1\right]}{\left[\frac{pqM'}{q^2 M} - \left(\frac{p}{q^2}\right)\left(\frac{pX'}{X}\right)\right]} = \frac{\left[\frac{pX'}{X} + 1\right]}{\left(\frac{\pi}{q}\right)\left[\frac{qM'}{M} - \frac{pX'}{X}\right]}
\end{aligned}$$

Then

$$\frac{\pi}{q}\left(\frac{dq}{d\pi}\right) = -\frac{(\varepsilon+1)}{(\varepsilon+\mu)} < 0 \tag{1.17}$$

where $\varepsilon \equiv \frac{pX'}{X} > 0$ is the price elasticity of exports and $\mu \equiv -\frac{qM'(q)}{M} > 0$ is the price elasticity of imports. From (1.17), we can state:

Lemma 1: *An increase in π always leads to a fall in the price of importables in terms of the non-traded goods, q, and therefore a fall in labor employment in sector M.*

Lemma 2: *An increase in π may result in an increase or a decrease in p, the relative price of exportables in terms of the non-traded goods. The necessary and sufficient condition for p to increase in response to an increase in π is that the price elasticity of imports demand exceeds unity: $\mu > 1$.*

Proof: From $p = q\pi$, we get

$$\frac{dp}{d\pi} = \pi\frac{dq}{d\pi} + q = q\left[\frac{\pi}{q}\frac{dq}{d\pi} + 1\right] = q\left[1 - \frac{(\varepsilon+1)}{(\varepsilon+\mu)}\right] = q\left[\frac{\mu-1}{\varepsilon+\mu}\right].$$

Thus

$$\frac{dp}{d\pi} > 0 \text{ iff } \mu > 1.$$

Then, in elasticity form,

$$\frac{\pi}{p}\frac{dp}{d\pi} = \frac{\mu-1}{\varepsilon+\mu}. \tag{1.18}$$

Remark 1: Since $pf'(L_X) = 1$, Lemma 2 shows that an increase in π will increase employment in sector X iff $\mu > 1$.

Remark 2: Let us find some sufficient conditions for $\mu > 1$. Recall that

$$\mu \equiv -\frac{q}{M}\frac{dM}{dq} = \frac{q}{M}\left[Q'_M(q) - D'_M(q)\right] > 0$$

Using (1.8), and (1.11), we have

$$\mu = \frac{q}{C_M - g(L_M(q))}\left[-\frac{1}{q^3 g''(L_M)} - \frac{1}{u''(C_M)}\right]$$

Example: Suppose $g(L_M) = BL_M - \frac{1}{2}L_M^2$ and $u(C_M) = AC_M - \frac{1}{2}C_M^2$. Then $g'' = -1 = u''$, and

$$\mu = \frac{q}{C_M - g(L_M(q))}\left[\frac{1+q^3}{q^3}\right].$$

In this case a sufficient condition for $\mu > 1$ is

$$\frac{q}{C_M(q) - g(L_M(q))} > 1$$

where

$$qg'(L_M) = 1 \rightarrow B - L_M = 1/q.$$

Then

$$\begin{aligned} g(L_M(q)) &= B\left(B - \frac{1}{q}\right) - \frac{1}{2}\left(B - \frac{1}{q}\right)^2 = \frac{1}{2}\left(B - \frac{1}{q}\right)\left[B + \frac{1}{q}\right] \\ &= \frac{1}{2}\left(B^2 - \frac{1}{q^2}\right) > 0 \text{ for } q > 1/B \end{aligned}$$

$$u'(C_M) = q \rightarrow C_M = A - q > 0 \text{ for } q < A$$

$$\frac{q}{C_M(q) - g(L_M(q))} = \frac{q}{A - q - \frac{B^2}{2} + \frac{1}{q^2}} > 0 \text{ if } q > 0 \text{ and } A - \frac{B^2}{2} > q - \frac{1}{2q^2}$$

Then , assuming $\frac{1}{B} < q < A$, $B > 1/A$, and $A - \frac{B^2}{2} > q - \frac{1}{2q^2}$, we have the condition

$$\frac{q}{C_M(q) - g(L_M(q))} \geq 1 \iff 2q - \frac{1}{2q^2} > A - \frac{B^2}{2}$$

e.g., for $q = A/2$, and $A > 2B > 2/A$, we have $\mu > 1$.

Effects of a trade shock on sectoral employments

Assume perfect labor mobility across the three sectors, the equilibrium labour allocation is depicted in a diagram (Figure A.1) where the curve on the left measures the value of marginal product of labour in the exportable sector, $pf'(L_X)$, while the curve on the right measures the value of marginal product of labor in the importable sector, $qg'(L_M)$. These curves cut the horizontal line $W = 1$ at points I and J, where J is to the right of I. The distance IJ measures the employment in the non-traded good sector.

The effects of an increase in π on sectoral employment can be represented by shifts in the curves $pf'(L_X)$ and $qg'(L_M)$. Then, assuming $\mu > 1$, an increase in π will shift the curve $pf'(L_X)$ up, and shift the curve $qg'(L_M)$ down. Thus, L_X increases and L_M decreases. What happens to employment in the non-traded good sector? Since

$$L_N = \overline{L} - L_X - L_M$$

we have

$$\frac{dL_N}{d\pi} = -\frac{dL_X}{d\pi} - \frac{dL_M}{d\pi}$$

Now, from $pf'(L_X) = 1$, we have

$$f'(L_X)dp + pf''(L_X)dL_X = 0$$

$$\frac{dL_X}{dp} = -\frac{f'}{pf''} = -\frac{1}{p^2 f''} > 0$$

Similarly

$$\frac{dL_M}{dq} = -\frac{1}{q^2 g''} > 0$$

Then

$$\frac{dL_X}{d\pi} = \frac{dL_X}{dp}\frac{dp}{d\pi} = -\frac{1}{p^2 f''} q \left[\frac{\mu - 1}{\varepsilon + \mu}\right] > 0 \text{ iff } \mu > 1$$

$$\frac{dL_M}{d\pi} = \frac{dL_M}{dq}\frac{dq}{d\pi} = \frac{1}{q^2 g''}\frac{(\varepsilon + 1)q}{(\varepsilon + \mu)\pi} < 0$$

Thus

$$\frac{dL_N}{d\pi} = \frac{q}{\varepsilon + \mu}\left[\frac{\mu - 1}{p^2 f''} - \frac{(\varepsilon + 1)}{\pi q^2 g''}\right]$$

$$= \frac{q}{(\varepsilon + \mu)p}\left[\frac{\mu - 1}{pf''} - \frac{(\varepsilon + 1)}{qg''}\right]$$

Thus, employment in the non-traded good sector may increase or decrease (it increases if $\mu \leq 1$).

2. Effects of trade shocks on wage inequality when labor is imperfectly mobile

In the preceeding section, we assume perfect mobility of labor, so that the wage rates in the three sectors are equalized. We now turn to the case where labor mobility is restricted in the short run.

Consider an initial situation where wages are equalized and are equal to unity, $W_X^0 = W_M^0 = W_N = 1$. At that initial equilibrium, employments in sectors X and M are denoted by L_X^0 and L_M^0.

Now, assume that there is a shock that increases π. Assume $\mu > 1$. Then the shock shifts the curve $pf'(L_X)$ up and shifts the curve $qg'(L_M)$ down, because p now takes a higher value, $p^* > p^0$, and q now takes a lower value, $q^* < q^0$.

Since labor cannot move across sectors in the short run, the higher marginal value product of labor in sector X results in a higher wage in that sector:

$$W_X = p^* f'(L_X^0) > p^0 f'(L_X^0) = W_X^0$$

Similarly

$$W_M = q^* g'(L_M^0) < q^0 g'(L_M^0) = W_M^0$$

The wage in the non-traded good sector remains unchanged, at $W_N = 1$.

The wage inequality gives workers an incentive to move from the low wage sectors to the higher wage sector. However, it takes time to move (e.g., workers need to be re-trained). How fast they can move depends on their training costs, which we assume to be dependent on their education level. Workers that had more years of schooling are presumably better equipped to learn new skills. We do not model schooling decisions here. We simply try to capture workers' heterogeneity by assuming sluggish labor mobility.

Let us assume that time is continuous and that the rate of labor outflow from a low wage sector to the highest wage sector (sector X) is proportional to the wage differential:

$$\frac{dL_M(t)}{dt} = \beta L_M(t) \times [W_M(t) - W_X(t)] < 0 \text{ for } W_M < W_X$$

$$\frac{dL_N(t)}{dt} = -\beta L_N(t) \times [W_N(t) - W_X(t)] < 0 \text{ for } W_N < W_X$$

where $\beta > 0$ is the speed of adjustment, which is a function of the average education level of the workforce.

Now, since the wage in each sector equals the value of the marginal product of labour in that sector, we have

$$W_X(t) = p^* f'(L_X(t))$$

$$W_M(t) = q^* g'(L_M(t))$$

$$W_N(t) = 1$$

Let us consider the differential equations

$$\frac{dL_N(t)}{dt} = \beta L_N(t)\left[1 - p^* f'(L_X(t))\right] = \beta\left[\overline{L} - L_X(t) - L_M(t)\right]\left[1 - p^* f'(L_X(t))\right]$$

$$\frac{dL_M(t)}{dt} = \beta L_M(t)\left[q^* g'(L_M(t)) - p^* f'(L_X(t))\right]. \tag{2.1}$$

Since $L_M + L_N + L_X = \overline{L}$, we deduce that

$$\frac{dL_X(t)}{dt} = -\frac{dL_M(t)}{dt} - \frac{dL_N(t)}{dt}$$

i.e.,

$$\frac{dL_X(t)}{dt} = \beta L_M(t)\left[1 - q^* g'(L_M(t))\right] + \beta\left[\overline{L} - L_X(t)\right]\left[p^* f'(L_X(t)) - 1\right]. \tag{2.2}$$

Then it can be shown that the system described by the pair of differential equations (2.1) and (2.2) has a steady state that is asymptotically stable. However, in this paper we are interested only in short run questions, for example, what happens to the wage gaps 5 periods after the shock?

3. An Example

Assume the demand functions are $D_X = A_X - p$ and $D_M = A_M - q$. Assume the production functions are

$$Q_X = f(L_X) = \frac{1}{\alpha}(L_X)^\alpha \text{ and } Q_M = g(L_M) = \frac{1}{\alpha}(L_M)^\alpha$$

Then the condition $pf'(L_X) = 1$ gives

$$L_X = p^{\frac{1}{1-\alpha}}.$$

It follows that

$$Q_X(p) = \frac{1}{\alpha} p^{\frac{\alpha}{1-\alpha}},$$

$$Q_M(q) = \frac{1}{\alpha} q^{\frac{\alpha}{1-\alpha}}.$$

The imports demand function is

$$M(q) = D_M(q) - Q_M(q) = A_M - q - \frac{1}{\alpha} q^{\frac{\alpha}{1-\alpha}}.$$

and the exports supply function is

$$X(p) = Q_X(p) - D_X(p) = \frac{1}{\alpha} p^{\frac{\alpha}{1-\alpha}} - A_X + p.$$

For simplicity, let $\alpha = 1/2$. Then

$$M(q) = A_M - 3q > 0 \text{ iff } q < A_M/3$$

$$X(p) = 3p - A_X > 0 \text{ iff } p > A_X/3$$

The price elasticity of imports demand is

$$\mu = -\frac{qM'(q)}{M} = \frac{3q}{A_M - 3q} > 0 \text{ for } M(q) > 0$$

and $\mu > 1$ iff $q > A_M/6$. In what follows, we consider q in the range

$$\frac{A_M}{6} < q < \frac{A_M}{3}$$

The price elasticity of export supply is

$$\varepsilon = \frac{3p}{3p - A_X} > 0 \text{ for } X(p) > 0$$

Recall that

$$\pi = \frac{p}{q} = \frac{P_X^*}{P_M^* \tau_M \tau_X}$$

The trade balance condition is $M(q) = \pi X(\pi q)$. This yields $A_M - 3q = \pi(3\pi q - A_X)$, i.e.,

$$3q(\pi^2 + 1) = A_M + \pi A_X.$$

Then

$$q = \frac{A_M + \pi A_X}{3(\pi^2 + 1)}. \tag{3.1}$$

Note that the restrictions that $q < A_M/3$ and $p > A_X/3$ imply the following restriction on π

$$\pi \geq \frac{A_X/3}{q} \geq \frac{A_X/3}{A_M/3} = \frac{A_X}{A_M} \text{ i.e. } \pi A_M > A_X \tag{3.2}$$

From (3.1)

$$\frac{dq}{d\pi} = \frac{1}{3}\left[\frac{A_X(\pi^2+1) - 2\pi(A_M + \pi A_X)}{(\pi^2+1)^2}\right]$$

$$= \frac{1}{3}\left[\frac{-A_X\pi^2 + A_X - 2\pi A_M}{(\pi^2+1)^2}\right] < 0 \text{ because } \pi A_M > A_X$$

We now assign some numerical values to the parameters A_M, A_X and π, and calculate the effect of shock (an increase in π) on the wage gaps, and how the wage gaps are reduced over a number of periods.

3.1. The initial equilibrium

Consider an initial situation where

$$A_M = 2,\ A_X = 1 \text{ and } \pi = \pi^r = 1$$

(where the superscript r indicate that this is the reference scenario). Then the initial equilibrium prices are

$$q^r = \frac{A_M + \pi A_X}{3(\pi^2+1)} = \frac{3}{6} = \frac{1}{2}$$

and

$$p^r = \pi q = \frac{1}{2}$$

Then

$$L_X^r = p^{\frac{1}{1-\alpha}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4},\ Q_X^r = 2p^r = 1$$

$$L_M^r = q^{\frac{1}{1-\alpha}} = \frac{1}{4},\ Q_M^r = 2q^r = 1$$

The value of national income is

$$\begin{aligned} Y &= \overline{L} + [pQ_X^r - L_X^r] + [qQ_M^r - L_M^r] \\ &= \overline{L} + \frac{1}{2} \end{aligned}$$

Domestic consumption of the goods are

$$C_X^r = A_X - p = 1 - \frac{1}{2} = \frac{1}{2}$$

$$C_M^r = A_M - p = 2 - \frac{1}{2} = \frac{3}{2}$$

Imports are

$$M(q) = \frac{3}{2} - 1 = \frac{1}{2}$$

Exports are

$$X(q) = 3p - A_X = \frac{1}{2}$$

Domestic consumers' total expenditure on the tradable goods are

$$pC_X + qC_M = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{2}\right) = 1$$

Assume that

$$\overline{L} = 1$$

Then national income is

$$Y = \overline{L} + \frac{1}{2} = 1.5$$

and thus the consumption of nontraded goods is

$$C_N = Y - (pC_X + qC_M) = 1.5 - 1 = 0.5$$

The initial labor allocations are $L_N = 0.5$, $L_X = 1/4$ and $L_M = 1/4$.

3.2. A trade shock

Now, consider an increase in π from its initial value of $\pi^r = 1$, e.g., caused by a fall in τ_X. The restriction $\pi > A_X/A_M$ still holds. Let the new π be denoted by π^*. Assume $\pi^* = 1.2$, i.e., the terms of trade increase by 20%.Then

$$q^* = \frac{A_M + \pi^* A_X}{3(\pi^{*2} + 1)} = \frac{2 + 1.2}{3\left((1.2)^2 + 1\right)} = 0.437\,16$$

and

$$p^* = \pi^* q^* = 0.524\,59$$

The new *long-run* equilibrium allocation of labour is given by

$$L_X^* = p^{*\frac{1}{1-\alpha}} = (0.524\,59)^2 = 0.275\,19$$

$$L_M^* = (q^*)^2 = (0.437\,16)^2 = 0.191\,11$$

and

$$L_N^* = \overline{L} - L_X^* - L_M^* = 1 - 0.275\,19 - 0.191\,11 = 0.533\,7$$

And the long-run equilibrium wages are

$$W_X^* = p^* f'(L_X^*) = (0.524\,59)\,\frac{1}{\sqrt{0.275\,19}} = 1$$

$$W_M^* = q^* g'(L_M^*) = (0.437\,16)\,\frac{1}{\sqrt{0.191\,11}} = 1$$

$$W_N^* = P_N^* = 1,$$

3.3. Short-run adjustments

In the short run, labor mobility across sectors is restricted. Immediately after the shock, labor allocation is still the same as at the initial equilibrium. Wages in the three sectors are equal to the value of the marginal product of labor:

$$W_X(t) = p^* f'(L_X(t)) = p^* L_X(t)^{\alpha-1} = p^* L_X(t)^{-1/2}$$

$$W_M(t) = q^* g'(L_M(t)) = q^* L_M(t)^{-1/2}$$

$$W_N(t) = 1$$

Then, using eqs (2.1) and (2.2),

$$\frac{dL_M(t)}{dt} = \beta L_M(t)\,[q^* g'(L_M(t)) - p^* f'(L_X(t))]$$

$$\frac{dL_X(t)}{dt} = \beta L_M(t)\left[1 - q^- g'(L_M(t))\right] + \beta\left[\overline{L} - L_X(t)\right]\left[p^+ f'(L_X(t)) - 1\right]$$

$$\dot{L}_M = \beta L_M\left[q^* L_M^{-1/2} - p^* L_X^{-1/2}\right]$$

$$\dot{L}_X = \beta L_M\left[1 - q^* L_M^{-1/2}\right] + \beta\left(\overline{L} - L_X\right)\left(p^* L_X^{-1/2} - 1\right)$$

Discrete-time approximation yields two difference equations:

$$L_M(t+1) = L_M(t) + \beta q^* L_M(t)^{1/2} - \beta p^* L_M(t) L_X(t)^{-1/2} \tag{3.3}$$

$$L_X(t+1) = L_X(t) + \beta L_M(t) - \beta q^* L_M(t)^{1/2} + \beta \overline{L} p^* L_X^{-1/2} - \beta \overline{L} - \beta p^* L_X^{1/2} + \beta L_X \tag{3.4}$$

With $L_M(0) = 0.25 = L_X(0), \overline{L} = 1$,and $p^* = 0.524\,59, q^* = 0.437\,16$.

Immediately after the shock, the labor allocation remains unchanged, and thus there is a big divergence in the wage rates. Denote by $W_X(0)$ and $W_M(0)$ the wage rates in industry X and industry M immediately after the shock:

$$W_X(0) = p^* f'(L_X^r) = 0.524\,59 \times \frac{1}{\sqrt{0.25}} = 1.\,049\,2$$

$$W_M(0) = q^* g'(L_M^r) = 0.437\,16 \times \frac{1}{\sqrt{0.25}} = 0.874\,32$$

The wage gap on impact is

$$G(0) \equiv W_X(0) - W_M(0) = 1.\,049\,2 - 0.874\,32 = 0.174\,88$$

The average wage is

$$W_X \frac{L_X}{\overline{L}} + W_M \frac{L_M}{\overline{L}} + W_N \frac{L_N}{\overline{L}} = \frac{1.\,049\,2}{4} + \frac{0.874\,32}{4} + \frac{1}{2} = 0.980\,88$$

Immediately after the initial shock, the ratio of the average wage of the top 20% wage earners to that of the bottom 20% wage earners is

$$\rho(0^+) = \frac{1.\,049\,2}{0.874\,32} = 1.2$$

Assume that $\beta = 0.05$. Using the difference equations (3.3) and (3.4), we compute the employment levels in industries X and M for five periods after the trade shock.

Note that $\beta q^* = (0.05)\,0.437\,16 = 0.02185\,8$ and $\beta p^* = (0.05)\,0.524\,59 = 0.02623$.

PERIOD 1:

$$L_M(1) = 0.25 + 0.02185\,8\sqrt{0.25} - 0.02623\left(\frac{0.25}{\sqrt{0.25}}\right) = 0.247\,81$$

i.e., a small outflow from sector M. The sector-M outflow rate in period 1 is

$$\frac{0.25 - 0.247\,81}{0.25} = 0.008\,76, \text{ i.e., less than } 1\%$$

$$L_X(1) = 0.25 - (0.05)\,(1 - 0.25 - 0.25) - 0.02185\,8\sqrt{0.25} + 0.02623\frac{1}{\sqrt{0.25}} - 0.02623\sqrt{0.25} \;= 0.253\,42$$

The sector-X inflow rate in period 1 is

$$\frac{0.253\,42 - 0.25}{0.25} = 0.013\,68\text{, i.e., about } 1.4\%$$

PERIOD 2:

$$L_M(2) = 0.247\,81 + 0.02185\,8\sqrt{0.247\,81} - 0.02623\frac{(0.247\,81)}{\sqrt{0.253\,42}} = 0.245\,78$$

$$L_X(2) = 0.256\,5$$

PERIOD 3:

$$L_M(3) = 0.245\,78 + 0.02185\,8\sqrt{0.245\,78} - 0.02623\frac{(0.245\,78)}{\sqrt{0.256\,5}} = 0.243\,89$$

$$L_X(3) = 0.259\,28$$

PERIOD 4:

$$L_M(4) = 0.243\,89 + 0.02185\,8\sqrt{0.243\,89} - 0.02623\frac{(0.243\,89)}{\sqrt{0.259\,28}} = 0.242\,12$$

$$L_X(4) = 0.261\,8$$

PERIOD 5 :

$$L_M(5) = 0.242\,12 + 0.02185\,8\sqrt{0.242\,12} - 0.02623\frac{(0.242\,12)}{\sqrt{0.261\,8}} = 0.240\,46$$

$$L_X(5) = 0.264\,08$$

So, after 5 periods, the wages are

$$W_M(5) = q^* g'(L_M(5)) = \frac{0.437\,16}{\sqrt{0.240\,46}} = 0.891\,50$$

and

$$W_X(5) = p^* f'(L_X(5)) = \frac{0.524\,59}{\sqrt{0.264\,08}} = 1.\,020\,8$$

The wage gap after 5 periods is

$$G(5) \equiv W_X(5) - W_M(5) = 1.\,020\,8 - 0.891\,50 = 0.129\,3$$

The ratio of the average wage of the top 20% wage earners to that of the bottom 20% wage earners is (after 5 periods of adjustments) is

$$\rho(5) = \frac{1.020\,8}{0.891\,50} = 1.145$$

This is to be compared with the initial impact effect,

$$\rho(0^{+}) = 1.2$$

Thus labour partial mobility leads in a small mitigation of the wage gap after 5 periods. The 5-period mitigation factor, defined as the percentage reduction in the wage gap, is

$$\frac{G(0^{+}) - G(5)}{G(0^{+})} = \frac{0.174\,88 - 0.129\,3}{0.174\,88} = 0.260\,64$$

3.4. What happens if labor mobility is higher?

Now, consider a higher coefficient of labor mobility, say $\beta = 0.1$. Then

$$\beta q^{*} = (0.1)\,0.437\,16 = 0.0437\,16$$

$$\beta p^{*} = (0.1)\,0.524\,59 = 0.0524\,59$$

PERIOD 1:

$$L_M(1) = 0.25 + 0.0437\,16\sqrt{0.25} - 0.0524\,59\frac{(0.25)}{\sqrt{0.25}} = 0.245\,63$$

$$L_X(1) = 0.256\,83$$

The sector-M outflow rate in period 1 is

$$\frac{0.25 - 0.245\,63}{0.25} = 0.017\,48 \text{ i.e., around } 1.7\%$$

The sector-X inflow rate in period 1 is

$$\frac{0.256\,83 - 0.25}{0.25} = 0.027\,32\text{, i.e., around } 2.7\%$$

PERIOD 2:

$$L_M(2) = 0.245\,63 + 0.0437\,16\sqrt{0.245\,63} - 0.0524\,59\frac{(0.245\,63)}{\sqrt{0.256\,83}} = 0.241\,87$$

$$L_X(2) = 0.262\,34$$

PERIOD 3:

$$L_M(3) = 0.241\,87 + 0.0437\,16\sqrt{0.241\,87} - 0.0524\,59\frac{(0.241\,87)}{\sqrt{0.262\,34}} = 0.238\,60$$

$$L_X(3) = 0.266\,81$$

PERIOD 4:

$$L_M(4) = 0.238\,60 + 0.0437\,16\sqrt{0.238\,60} - 0.0524\,59\frac{(0.238\,60)}{\sqrt{0.266\,81}} = 0.235\,72$$

$$L_X(4) = 0.270\,46$$

PERIOD 5:

$$L_M(5) = 0.235\,72 + 0.0437\,16\sqrt{0.235\,72} - 0.0524\,59\frac{(0.235\,72)}{\sqrt{0.270\,46}} = 0.233\,17$$

$$L_X(5) = 0.273\,44$$

Recall $p^* = 0.524\,59, q^* = 0.437\,16$.The wages in period 5 are

$$W_M(5) = q^* g'(L_M(5)) = 0.905\,32$$

$$W_X(5) = \frac{0.524\,59}{\sqrt{0.273\,44}} = 1.\,003\,2$$

The wage gap in period 5 is

$$1.\,003\,2 \;-\; 0.891\,50 = 0.111\,7$$

As expected, a higher mobility rate implies a mitigation of the wage gap. The 5-period mitigation factor, defined as the percentage reduction in the wage gap, is

$$\frac{G(0^+) - G(5)}{G(0^+)} = \frac{0.174\,88 - 0.111\,7}{0.174\,88} = 0.361\,28$$

The inequality index in period 5, defined as the ratio of the income of the top 20% wage earners to bottom 20% wage earners, is

$$\rho(5) = \frac{1.\,003\,2}{0.891\,50} = 1.\,125\,3$$

(compared with $1.\,145$ for $\beta = 0.05$). As expected, the higher labor mobility implies a lower degree of wage inequality.